PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO
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Uma abordagem sobre Sistemas de Amortização à luz da Educação Financeira

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Resumo

Este artigo apresenta um recorte de uma pesquisa de mestrado acerca do ensino de Empréstimos & Financiamentos na Educação Básica. Nosso objetivo aqui é apresentar uma proposta de ensino dos principais sistemas de amortização presentes no mercado brasileiro. Este tópico de ensino está inserido nas discussões acerca da Educação Financeira Escolar, a qual apresentamos sua importância hoje para a sociedade e sua implantação que vem ocorrendo a partir da homologação da BNCC. Nesse cenário, apresentamos um estudo exemplificado e comparativo dos sistemas de amortização mais usuais, versando também sobre a presença desses em livros didáticos e em dissertações do PROFMAT. Em seguida, narramos a confecção, a aplicação e a análise do produto educacional construído ao longo da pesquisa: um livro construído pelos autores desse texto para auxiliar professores no ensino dos sistemas SAC e Table Price. O artigo se encerra na expectativa de promover conhecimentos que influenciem o comportamento de uma nova geração de consumidores, contribuindo para a Educação Financeira dos leitores.

Palavras-chave: BNCC. Educação Financeira. Ensino Médio. Sistemas de Amortização. Table Price e SAC.

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An approach to amortization systems in light of Financial Education

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Abstract

This article presents an excerpt from a master's research on the teaching of Loans & Financing in Basic Education. Our objective here is to present a teaching proposal on the main amortization systems present in the Brazilian market. This teaching topic is part of the discussions about School Financial Education, which we present its importance today for society and its implementation that has been taking place since the approval of the BNCC. In this scenario, we present an exemplified and comparative study of the most common amortization systems, also dealing with their presence in textbooks and PROFMAT dissertations. Then, we narrate the confection, application and analysis of the educational product built during the research: a book built by the authors of this text to assist teachers in teaching the SAC and Price Table systems. The article ends with the expectation of promoting knowledge that influences the behavior of a new generation of consumers, contributing to the Financial Education of the readers.

Keywords: BNCC. Financial education. High school. Amortization Systems. Price and SAC table.

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Un enfoque de los sistemas de amortización a la luz de la Educación Financiera

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Resumen

Este artículo presenta un extracto de una investigación de maestría sobre la enseñanza de Préstamos y Financiamiento en la Educación Básica. Nuestro objetivo aquí es presentar una propuesta didáctica sobre los principales sistemas de amortización presentes en el mercado brasileño. Este tema didáctico forma parte de las discusiones sobre Educación Financiera Escolar, que presentamos su importancia hoy para la sociedad y su implementación que se viene dando desde la aprobación de la BNCC. En este escenario, presentamos un estudio ejemplificado y comparativo de los sistemas de amortización más comunes, tratando también su presencia en los libros de texto y disertaciones del PROFMAT. Luego, narramos la confección, aplicación y análisis del producto educativo construido durante la investigación: un libro construido por los autores de este texto para auxiliar a los docentes en la enseñanza de los sistemas SAC y Tabla de Precios. El artículo finaliza con la expectativa de promover conocimientos que influyan en el comportamiento de una nueva generación de consumidores, contribuyendo a la Educación Financiera de los lectores.

Palabras clave: BNCC. Educación financiera. Escuela secundaria. Sistemas de amortización. Precio y tabla SAC.

Introduction

According to the National Common Curricular Base (BNCC), published in 2018, young people completing the final stage of Basic Education, the High School level, must be able to develop their life projects with critical thinking, autonomy, and responsibility. In part, the fulfillment of these life projects is related to the students' consumption behaviors, as items such as courses, vehicles, and real estate are often part of these projects.

For the payment of the aforementioned items, when not made in full, some young people will encounter situations involving financing and/or loans. In this regard, this group requires strategies to help them make conscious and responsible decisions regarding the acquisition of goods and services. Such strategies can be learned through knowledge of "Interest Rates" and "Progressions," mathematical topics commonly addressed in High School textbooks. This learning should be conducted under the focus of Financial Education, a transversal theme made mandatory by the BNCC as of 2020.

Given this context, we conducted a Master's research on the teaching of Loans & Financing in Basic Education, aiming to develop suitable approaches for high school students on Amortization Systems and, from these, create a teaching proposal for educators working in this segment. Thus, the research, developed and completed within the scope of the Professional Master's Program in Mathematics in the National Network – PROFMAT, at the Federal Center for Technological Education of Minas Gerais – CEFET-MG, in Belo Horizonte, aims to contribute to the Financial Education of students and teachers, raising awareness about the advantages and disadvantages of installment purchases.

Therefore, in this article, we present excerpts from this research, focusing on the theoretical foundation regarding the relevance of Financial Education and its implementation in Basic Education. Next, we present an exemplary and comparative study of the main amortization systems, to which we associate the development of the Loans & Financing topic. Finally, we describe the results of the research, namely an educational product developed for teaching the main financing modalities: SAC and Price Table. In conclusion, we present our studies on Financial Education.

School-Based Financial Education

In this article, Financial Education is defined as a process capable of guiding a financially illiterate individual toward financial literacy (FREITAS & MOREIRA, 2020). While the term financial illiteracy refers to a person's lack of knowledge regarding basic financial matters—such as managing a household and/or personal budget, comparing prices, calculating interest, evaluating payment methods for purchases made on credit or in installments, and creating savings plans—financial literacy consists of "the combination of awareness, knowledge, skills, attitudes, and behaviors necessary to make sound financial decisions and ultimately achieve individual financial well-being" (EIOPA, 2011, p.10).

The financially illiterate individuals referenced in this article are identified by the Organisation for Economic Co-operation and Development (OECD, 2005) as vulnerable consumers. These include individuals excluded from the financial system, indebted young people, and workers who rely on government assistance to sustain themselves during retirement. In addition to compromising their quality of life and hindering investments in key areas such as higher education, housing finance, and retirement, the poor financial habits of these consumers affect not only themselves and their families but also the broader society in which they live.

Within this context, financial illiteracy can be characterized as a social issue, and the process associated with Financial Education as a form of mobilization against this problem offering a range of opportunities to each of the aforementioned groups.

Financial Education aims to provide consumers excluded from the financial system particularly low-income individualswith the opportunity to participate in that system. Once financially literate, they would avoid high transaction costs and learn to save. As for workers, they would benefit from the information and skills necessary to make secure choices regarding pension plans and/or individual savings to ensure a comfortable retirement. Newly employed young people, once financially literate, would be capable of managing their budgets and controlling their debts, enabling them to direct their resources toward the acquisition of major assets such as vehicles, real estate, and education.

The above discussion illustrates the usefulness of Financial Education both for individuals and for the government. By developing autonomy in decision-making that contributes to a better quality

of life and a dignified retirement, individuals become less dependent on public pension funds. According to Silva and Powell (2015), achieving financial literacy across an entire population yields benefits not only for individuals but also for society as a whole, as informed decision-making and the demand for high-quality services stimulate competition and market innovation.

In addition to the aforementioned advantages, Financial Education also fosters a better understanding of basic financial information, enabling consumers with capital to invest classified by the OECD as non-vulnerable to make sound transactions and investments.

Having outlined the needs and benefits of promoting financial literacy among individuals in society, we now turn to how Financial Education can be implemented. The approach may vary depending on which segment of the population is being targeted. Since the focus of this article is on students in Basic Education, we present below a definition of School-Based Financial Education.

> School-Based Financial Education consists of a set of informational resources through which students are introduced to the world of money and encouraged to develop an understanding of finance and economics. This is achieved through a teaching process that enables them to analyze, make informed judgments, take decisions, and adopt critical stances on financial issues that affect their personal lives, their families, and the society in which they live (SILVA e POWELL, 2013, p. 12).

OECD reports emphasize the need to incorporate Financial Education into schools, either as an elective subject or, more assertively, as a mandatory component of the official school curriculum. According to the analysis by Silva and Powell (2015), the rationale for this lies in the school environment's ability to reach, almost entirely, the target audience: children and young people. Once educated, these individuals can become effective disseminators of behaviors that help shape a stronger financial culture across the broader population.

In Brazil, the starting point for implementing Financial Education in schools was marked by Decree No. 5,685/2006, which established the Committee for the Regulation and Oversight of Financial, Capital, Insurance, Pension, and Capitalization Markets (Coremec). This committee was tasked with promoting and coordinating the implementation of a nationwide strategy for Financial Education, defining elements such as the target audience, objectives, and areas of action.

Throughout various discussions, Coremec defined initiatives aimed at Basic Education students to encourage the habit of saving. These initiatives laid the groundwork for the National Strategy for Financial Education (ENEF), enacted through Decree No. 7,397/2010. The same decree also established the National Committee for Financial Education (CONEF) and the Pedagogical Support Group (GAP) as entities responsible for supporting the implementation of Financial and Pension Education across the national territory. Since then, several initiatives have been developed within the school setting to consolidate Financial Education in Basic Education, culminating in the implementation of the National Common Curricular Base (BNCC) in 2018.

The National Common Curricular Base (BNCC) is the document that defines the set of essential and indispensable learning objectives for students in public and private education networks throughout the country. It also regulates all interdisciplinary knowledge and practices that must be addressed in Basic Education, including the discussion of contemporary issues that impact various aspects of human life. Among these topics, Financial Education is highlighted and must be approached in a transversal and integrative manner.

In the case of Mathematics, knowledge is organized into five thematic units: Numbers; Algebra; Geometry; Quantities and Measurements; and Probability and Statistics. References to Financial Education appear in the very first unit.

> Another aspect to be considered in this thematic unit is the study of basic concepts in economics and finance, aimed at promoting students' financial education. Thus, topics such as interest rates, inflation, financial investments (return and liquidity of an investment), and taxes can be discussed. This thematic unit encourages an interdisciplinary approach that encompasses cultural, social, political, and psychological dimensions, in addition to the economic dimension, in addressing issues related to consumption, labor, and money (BRASIL, 2018, p. 269, emphasis added).

In this context, it is within the Mathematics classroom that Financial Education will find its greatest space, as certain mathematical concepts—such as percentages and interest—can serve as starting points for discussions on profit and loss, inflation rates, interest rates, and installment purchases. The facilitation of these discussions can extend into other areas of knowledge, equipping students with the ability to analyze and reflect, thereby supporting them in making informed decisions

about the world of work and responsible consumption.

Within this framework, we now analyze a set of competencies outlined by the BNCC for high school students, aimed at developing Financial Education through Mathematics instruction. The development of these competencies, presented in Table 1, directly addresses the needs of the vulnerable consumers previously mentioned.

Through EM13MAT303, low-income individuals would learn to avoid high-cost financial transactions, while EM13MAT203 would enable young people newly entering the workforce and their families to appropriately use digital tools for managing their budgets. Furthermore, competency EM13MAT104 would assist students and workers in making informed choices regarding loans, pension plans, and/or individual savings, given that the key element in these financial modalities is the interest rate applied.

Box 1 - BNCC Competencies Contextualized to Financial Education

SKILL	DESCRIPTION
EM13MAT101	Critically interpret economic and social situations and facts related to Natural Sciences involving the variation of quantities, through the analysis of graphs of represented functions and rates of change, with or without the use of digital technologies.
EM13MAT104	Interpret rates and indices of socioeconomic nature (human development index, inflation rates, among others), investigating the calculation processes of these numbers, to critically analyze reality and produce arguments.
EM13MAT106	Identify everyday life situations in which it is necessary to make choices considering probabilistic risks (using one or another contraceptive method, opting for one medical treatment over another, etc.).
EM13MAT203	Apply mathematical concepts in planning, execution and analysis of actions involving the use of applications and spreadsheet creation (for family budget control, interest calculations—simple and compound—among others), to support decision-making.
EM13MAT303	Interpret and compare situations involving simple interest with those involving compound interest, through graphical representations or spreadsheet analysis, highlighting the linear or exponential growth in each case.

Source: BRASIL (2018, p.533,534,536)

Given this, we will draw on these competencies to explore the teaching of Loans & Financing, considering the relevance of this topic within the discussions on Financial Education proposed by the BNCC. In this context, we associate the topic with installment purchases, whose payment strategies are examined through Amortization Systems.

Introducing Amortization Systems

As previously stated, one of the goals of School-Based Financial Education is to develop individuals who are "capable of analyzing, making informed judgments, making decisions, and adopting critical stances on financial matters" (SILVA & POWELL, 2013, p. 12). In this context, the focus of this section is on decisions related to installment purchases, to which we associate the topic of Loans & Financing.

In light of the objectives established by the BNCC for High School, the teaching of Loans & Financing effectively addresses the four main goals set forth in the document for this educational stage. These are: (i) consolidation and deepening of knowledge; (ii) focus on the world of work and citizenship; (iii) ethical development, intellectual growth, and critical thinking; and (iv) understanding of scientific and technological foundations.

According to Pereira and Couto (2017), the topic in question serves as a deepening of the concepts of percentages and interest rates introduced in Elementary School (i), and also guides students who wish to pursue careers in the banking or financial sectors (ii). This content, by relating theory and practice in the teaching of Mathematics with the aid of software and spreadsheets (iv) leads students to the autonomy and critical awareness needed to avoid being misled by false advertising, whether in retail or various financial institutions (iii).

Within this context, working with Loans & Financing also aims to develop the competencies presented in Table 1 of the previous section. After all, interest rates play a key role in the analysis of a financing contract (EM13MAT104), as do the installment amounts—whether fixed or variable. In both cases, the value of the installments and the outstanding balance over time must be observed, which can be done through the construction of electronic spreadsheets (EM13MAT203). Spreadsheet simulations and the graphs they generate allow for the analysis of the behavior of the variables involved in each process, including the types of interest applied to each financing model, thereby facilitating comparisons between available options (EM13MAT303).

Decisions regarding the most appropriate financing strategies are based on the analysis of various elements previously mentioned in this section, such as the interest rate and the number of installments, as well as the financing modality adopted. This set of elements is referred to as

amortization systems.

According to Freitas (2021), amortization is understood as the process of reducing a debt, until its full settlement, through monthly installment payments, in accordance with the system defined in the contract, to which interest on the outstanding balance is added. The parties involved in the negotiation must agree on the terms and interest rates, as well as on how the principal amount will be repaid. In this sense, each payment made during the process serves a dual purpose: to repay part of the original debt and to pay interest on the outstanding balance, which gradually decreases until it is fully extinguished.

The way in which the debt is repaid and the interest is paid along with how these components evolve over time defines the amortization systems. In these systems, the installment amount is calculated as the sum of the amortized principal and the interest on the outstanding balance. The distinction between the basic amortization systems lies in the constancy of the interest payments, the amortization amounts, or the installment values. Respectively, we are referring to the AMERICAN, SAC, and PRICE systems, which will be discussed below.

The American Amortization System referred to here simply as the American System is a process in which the debt amortization occurs in a single payment at the end of the term. Prior to that payment, the lender and borrower must agree on how the interest will be paid: either periodically throughout the contract or capitalized and paid at the end.

To illustrate how this system works, let us consider a debt of R$ 12,000.00, contracted at a monthly interest rate of 1%, to be paid at the end of 12 months. According to the specifications of this modality, the loan amount will be settled at the end of the final month. As previously mentioned, the interest accrued in the negotiation may be capitalized over the term or paid periodically.

In the first scenario, the capitalized interest is paid together with the original amount, generating a total that can be calculated using the well-known compound interest formula, $M = C \cdot (1 + i)^t$. Sendo assim, temos $M = 12000{,}00 \cdot (1 + 0{,}01)^{12} = R\$\, 13521{,}90$. In this case, no payments will be made throughout the contract term neither on the interest nor on the principal. The balance will be amortized in a single payment of R$ 12,000.00 at the end of the contract, with R$ 1,521.90 in interest accrued over the period.

In the second scenario, amortization still occurs only at the end of the agreement, but monthly installments are made to cover the interest on the loan amount. The progression of these payments is shown in Table 1.

Table 1 - American Amortization System

American Amortization System					
Month	Previous Debt	Fees	Amortization	Installment	Current Debt
1	R$ 12.000,00	R$ 120,00	R$ -	R$ 120,00	R$ 12.000,00
2	R$ 12.000,00	R$ 120,00	R$ -	R$ 120,00	R$ 12.000,00
3	R$ 12.000,00	R$ 120,00	R$ -	R$ 120,00	R$ 12.000,00
4	R$ 12.000,00	R$ 120,00	R$ -	R$ 120,00	R$ 12.000,00
5	R$ 12.000,00	R$ 120,00	R$ -	R$ 120,00	R$ 12.000,00
6	R$ 12.000,00	R$ 120,00	R$ -	R$ 120,00	R$ 12.000,00
7	R$ 12.000,00	R$ 120,00	R$ -	R$ 120,00	R$ 12.000,00
8	R$ 12.000,00	R$ 120,00	R$ -	R$ 120,00	R$ 12.000,00
9	R$ 12.000,00	R$ 120,00	R$ -	R$ 120,00	R$ 12.000,00
10	R$ 12.000,00	R$ 120,00	R$ -	R$ 120,00	R$ 12.000,00
11	R$ 12.000,00	R$ 120,00	R$ -	R$ 120,00	R$ 12.000,00
12	R$ 12.000,00	R$ 120,00	R$ 12.000,00	R$ 12.120,00	R$ -

Source: Freitas (2021)

In Table 1, it is evident that monthly interest payments, combined with zero amortization, keep the debt unchanged until the beginning of the 12th month, at the end of which a final installment of R$12,120.00 is paid this amount corresponds to the interest for the last period added to the principal amortization. In this option, with fixed interest rates, the sum of the “Installment” column (R$13,440.00) reveals that it is the more advantageous alternative, as it represents a lower total amount compared to that calculated in the first scenario (R$13,521.90).

In general terms, the American Amortization System stipulates that a principal amount V, financed over n months at a monthly interest rate i, will result in a total amount M given by $M = V \cdot (1 + i)^n$, if interest is capitalized, or $M = V \cdot (1 + n \cdot i)$, in the case of monthly interest payments. These formulas appear in high school textbooks in chapters on Financial Mathematics, when addressing compound and simple interest systems, respectively; however, they are usually presented in contexts unrelated to this specific financing model.

According to Bagatini (2010), the American System is a suitable option for individuals looking to start their own business, with the expectation of raising sufficient funds in the short term to repay the loan. However, due to the total amount payable, this amortization system is appropriate only for entrepreneurs who do not have a stable income to support installment-based debt repayment.

Conversely, for those who are able to make monthly payments in addition to interest, there are more advantageous systems in which the total amount to be paid is lower than that of the American System. These include the Constant Amortization System (SAC) and the French Amortization System (commonly known as the Price System), whose monthly payments serve, among other purposes, to reduce the outstanding balance either through constant amortization amounts or fixed installments, respectively.

The Constant Amortization System (SAC) is a method in which the outstanding balance decreases linearly through fixed amortization payments, calculated by dividing the financed amount by the number of installments. To each of these amortization payments, interest on the remaining debt is added, resulting in decreasing and linearly declining installments over the repayment period.

Unlike the American System, this new modality is one of the most common amortization systems in Brazil, particularly applied to real estate financing. Research by Santos (2015) and Marques (2016) highlights the use of this system in the operations of the Housing Finance System (SFH), in the Federal Government's "Minha Casa, Minha Vida" program, as well as in real estate loans offered by commercial banks and in transfers to private companies through government agencies.

To illustrate the evolution of debt in a SAC system, let us once again consider a debt of R$12,000.00, contracted at a monthly interest rate of 1%, to be paid over the course of one year. In this modality, the debt will be amortized in 12 installments of R$1,000.00, to which interest calculated on the outstanding balance is added. The steps of this arrangement are recorded in Table 2.

In Table 2, we observe that the amortization payments, fixed at R$1,000.00 throughout the entire financing period, reduce the outstanding balance linearly and, consequently, the interest calculated each month. In all columns except for "Month" and "Amortization" we identify values forming decreasing arithmetic progression.

Table 2 - Constant Amortization System (SAC)

Constant Amortization System					
Month	Previous Debt	Fees	Amortization	Installment	Current Debt
1	R$ 12.000,00	R$ 120,00	R$ 1.000,00	R$ 1.120,00	R$ 11.000,00
2	R$ 11.000,00	R$ 110,00	R$ 1.000,00	R$ 1.110,00	R$ 10.000,00
3	R$ 10.000,00	R$ 100,00	R$ 1.000,00	R$ 1.100,00	R$ 9.000,00
4	R$ 9.000,00	R$ 90,00	R$ 1.000,00	R$ 1.090,00	R$ 8.000,00
5	R$ 8.000,00	R$ 80,00	R$ 1.000,00	R$ 1.080,00	R$ 7.000,00
6	R$ 7.000,00	R$ 70,00	R$ 1.000,00	R$ 1.070,00	R$ 6.000,00
7	R$ 6.000,00	R$ 60,00	R$ 1.000,00	R$ 1.060,00	R$ 5.000,00
8	R$ 5.000,00	R$ 50,00	R$ 1.000,00	R$ 1.050,00	R$ 4.000,00
9	R$ 4.000,00	R$ 40,00	R$ 1.000,00	R$ 1.040,00	R$ 3.000,00
10	R$ 3.000,00	R$ 30,00	R$ 1.000,00	R$ 1.030,00	R$ 2.000,00
11	R$ 2.000,00	R$ 20,00	R$ 1.000,00	R$ 1.020,00	R$ 1.000,00
12	R$ 1.000,00	R$ 10,00	R$ 1.000,00	R$ 1.010,00	R$ -

Source: Freitas *et al* (2021)

Once the first terms (a_1) and the ratios *(r)* of these progressions are identified, the well-known general formula for an arithmetic progression (PA), $a_n = a_1 + r \cdot (n-1)$, allows us to express the following equations $j_n = 120 - 10 \cdot (n-1)$, $p_n = 1120 - 10 \cdot (n-1)$ e $D_n = 11000 - 1000 \cdot (n-1)$, which describe the evolution of the variables "Interest," "Installment," and "Outstanding Debt," respectively. Based on these expressions, it is possible to calculate the interest, the installment amount, and the outstanding balance for any given month , with $n \in IN$, which facilitates monitoring and control in long-term financial agreements.

In addition to the expressions mentioned above, we can adapt the formula for the sum of the first *n* terms of PA, $S_n = (a_1 + a_n) \cdot \frac{n}{2}$, to derive the expression. $M_n = (p_1 + p_n) \cdot \frac{n}{2}$, which is particularly useful for calculating the total amount paid up to a given month. Thus, the total amount paid over the course of the financing presented in Table 2 can be calculated either by summing the values in the "Installment" column or by applying the above formulas. For example, for *n = 12*, we have:

- $p_{12} = 1120 - 10 \cdot (12-1) = R\$1010{,}00$ e
- $S_{12} = (p_1 + p_{12}) \cdot \frac{12}{2} = (1120 + 1010) \cdot 6 = R\$12780{,}00.$

With regard to total amounts paid, it is evident that the SAC system is more advantageous than

the American System, as calculated from Table 1 (R$ 13,440.00). This outcome is not limited to the examples cited in this text. Mathematical generalizations for a debt V, financed over n months at a monthly interest rate i, allow us to conclude that, given these fixed parameters, the total amounts paid under the SAC system are consistently more favorable compared to other financing modalities.

A key recommendation when dealing with the SAC system concerns its use in real estate financing. In this context, Almeida (2019) warns of the importance of paying attention not only to lower interest rates but also to the additional costs related to insurance fees and other financing expenses, which increase the total cost of credit. Furthermore, Filgueiras (2019) notes that some financial institutions only approve loans whose installments correspond to a maximum of 30% of the household income. For many families, the high value of the initial installments under the SAC system exceeds this threshold. As such, an alternative to this drawback is the French Amortization System, which features fixed installments throughout the entire financing period.

The French Amortization System, commonly known as the Price Table, is based on a compound interest regime with fixed and periodic payments. Similar to SAC, the installments in the Price Table serve the dual purpose of amortizing the debt and paying interest on the outstanding balance. However, as a result of the constancy of the installment amounts, the portion of each payment allocated to amortization increases over time, while the interest portion decreases throughout the process.

With installment amounts set below the initial payments required by the SAC system, the Price Table presents a viable option for short- and medium-term financing. In Brazil, this modality is widely used for the financing of consumer goods in general, such as electronic devices, furniture, and automobiles, as well as for credit card bill installments and short-term loans. According to Silvestre (2015), the use of this system often goes unnoticed in everyday purchases, as most sellers and consumers are unfamiliar with the specific characteristics of the Price Table. The calculations are handled by the financial institutions that underwrite such transactions—credit card companies, banks, and credit cooperatives.

The elements of a financing arrangement governed by the Price Table, as well as their evolution over time, are presented in Table 3, which is based on an initial debt of R$12,000.00 amortized in 12 installments of R$1,066.19. For each payment—whose amount will be justified later in this section—

the amortization value is calculated as the difference between the fixed installment and the interest, which is 1% of the outstanding balance.

Unlike what was observed in the SAC system, Table 3 shows that the evolution of the elements in the Price Table does not follow arithmetic or geometric progressions—except for the "Amortization" column, which increases in a geometric progression with a common ratio of 1.01, and the "Installment" column, which may be viewed as a constant geometric or arithmetic progression. As previously described, the interest payments gradually decrease due to the reduction of the outstanding balance. Moreover, with fixed installments, decreasing interest implies increasing amortization.

Table 3 - French Amortization System (Price Table)

French Amortization System (Price Table)					
Month	Debit Balance	Fees	Amortization	Installment	Current Balance
1	R$ 12.000,00	R$ 120,00	R$ 946,19	R$ 1.066,19	R$ 11.053,81
2	R$ 11.053,81	R$ 110,54	R$ 955,65	R$ 1.066,19	R$ 10.098,17
3	R$ 10.098,17	R$ 100,98	R$ 965,20	R$ 1.066,19	R$ 9.132,96
4	R$ 9.132,96	R$ 91,33	R$ 974,86	R$ 1.066,19	R$ 8.158,11
5	R$ 8.158,11	R$ 81,58	R$ 984,60	R$ 1.066,19	R$ 7.173,50
6	R$ 7.173,50	R$ 71,74	R$ 994,45	R$ 1.066,19	R$ 6.179,05
7	R$ 6.179,05	R$ 61,79	R$ 1.004,39	R$ 1.066,19	R$ 5.174,66
8	R$ 5.174,66	R$ 51,75	R$ 1.014,44	R$ 1.066,19	R$ 4.160,22
9	R$ 4.160,22	R$ 41,60	R$ 1.024,58	R$ 1.066,19	R$ 3.135,64
10	R$ 3.135,64	R$ 31,36	R$ 1.034,83	R$ 1.066,19	R$ 2.100,81
11	R$ 2.100,81	R$ 21,01	R$ 1.045,18	R$ 1.066,19	R$ 1.055,63
12	R$ 1.055,63	R$ 10,56	R$ 1.055,63	R$ 1.066,19	-R$ 0,00

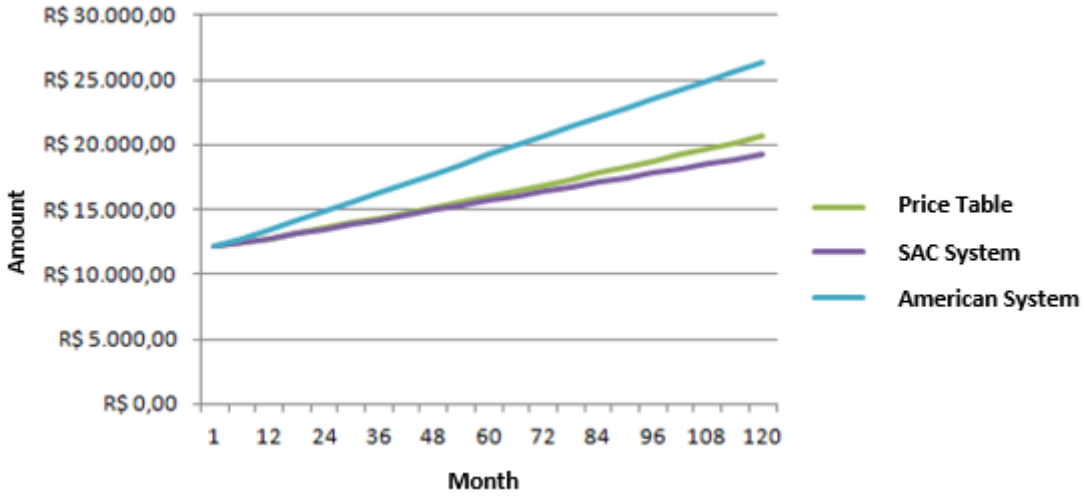
Source: Freitas *et al* (2021)

Given that the values in the "Amortization" column increase in a geometric progression (PG) with a common ratio q = 1,01, the expressions $A_n = 946{,}19 \cdot (1{,}01)^{n-1}$ and $S_n = 94619{,}00 \cdot [(1{,}01)^n - 1]$ can be used to calculate, respectively, the amortization amount in month n and the total amount repaid up to that month. These expressions are derived from the well-known formulas for geometric progressions - PG, $a_n = a_1 \cdot q^{n-1}$ e $S_n = a_1 \cdot \left(\frac{q^n - 1}{q-1}\right)$, used to determine the value of the n-th term and the sum of the first n elements of the sequence, respectively. Based on these formulas, we may also use the expression. $D_n = 12000{,}00 - 94619{,}00 \cdot [(1{,}01)^n - 1]$ to calculate the current outstanding balance, which is equivalent to the initial debt minus the total amount amortized.

With regard to the constancy of the installments, the amount of each payment (P) is calculated based on the loan amount (V), the interest rate (i), and the number of installments (n); variables which, in the Price Table, are related by the formula $P = \frac{V \cdot i}{1-(1+i)^{-n}}$. Returning to the scenario presented in Table 3, and using the set *(V, i, n) = (12000, 1%, 12)*, we have $P = \frac{12000 \cdot 0{,}01}{1-(1+0{,}01)^{-12}} = R\$1066{,}19$.

Regarding the total amount of installments in both systems—calculated either by summing the values in the "Installment" column of Tables 2 and 3 or by using the formulas previously discussed—the results shown in Table 3 (12 × R$1,066.19 = R$12,794.08) reveal a slight disadvantage of the Price Table compared to the SAC (R$12,780.00), as previously calculated. Once again, this observation is not limited to the examples presented. When the loan amount, term, and interest rate are fixed, mathematical generalizations lead to the conclusion that the total amount paid under a SAC-based financing arrangement will always be lower than that paid under a Price Table scheme. Figure 1, based on a R$12,000.00 loan at a monthly interest rate of 1%, illustrates this point graphically.

Figure 1 - Amortization Systems and Their Total Amounts

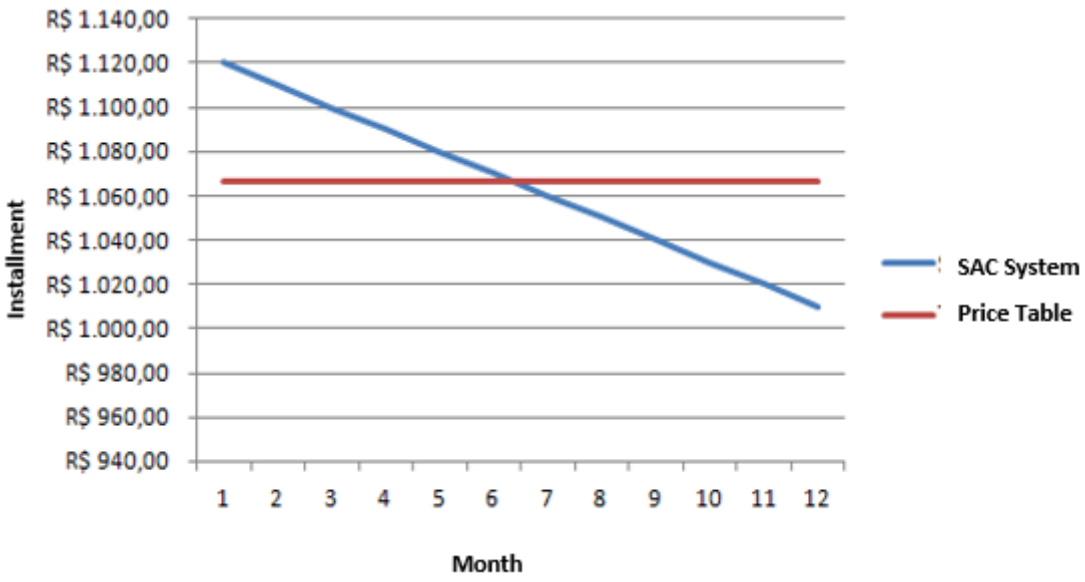


Source: Freitas (2021)

While the total amount paid stands as the main advantage of the SAC system over other financing options, the higher initial installment values in this modality represent a drawback especially when compared to the Price Table, which begins with lower payments. However, since SAC installments decrease over time, there will always come a point at which they become lower

than the fixed installments of the French system. Figure 2, constructed from the "Installment" columns of Tables 2 and 3, illustrates the evolution of payments over the financing period.

Figure 2 - Amortization Systems and Their Installments



Source: Freitas (2021)

In contrast to the high initial installment amounts of the SAC system, the Price Table remains the most widely used financing modality in the country. According to Filgueiras (2019), the justification for this preference lies in the constancy of the installments, which facilitates short- and medium-term monthly budgeting for borrowers (loan recipients). On the other hand, the Price Table also benefits the lender (the entity granting the loan), who ultimately receives a higher total amount another factor contributing to the preference for the French system.

An alternative to the high initial payments of the SAC system and the higher total amounts of the Price Table is the Growing Amortization System (SACRE). In this modality also known as a mixed system—the monthly installment corresponds to the arithmetic mean of the payments calculated under both systems for the given month, resulting in linearly decreasing monthly payments. Similar to the SAC system, each SACRE installment covers interest on the outstanding balance, which decreases over time, and amortizes increasingly larger portions of the original debt. Table 4 illustrates the progression of the variables involved in this type of financing.

Table 4 - Growing Amortization System (SACRE)

Growing Amortization System (SACRE)					
Month	Outstanding Balance	Fees	Amortization	Installment	Current Balance
1	R$ 12.000,00	R$ 120,00	R$ 973,09	R$ 1.093,09	R$ 11.026,91
2	R$ 11.026,91	R$ 110,27	R$ 977,82	R$ 1.088,09	R$ 10.049,08
3	R$ 10.049,08	R$ 100,49	R$ 982,60	R$ 1.083,09	R$ 9.066,48
4	R$ 9.066,48	R$ 90,66	R$ 987,43	R$ 1.078,09	R$ 8.079,05
5	R$ 8.079,05	R$ 80,79	R$ 992,30	R$ 1.073,09	R$ 7.086,75
6	R$ 7.086,75	R$ 70,87	R$ 997,23	R$ 1.068,09	R$ 6.089,53
7	R$ 6.089,53	R$ 60,90	R$ 1.002,20	R$ 1.063,09	R$ 5.087,33
8	R$ 5.087,33	R$ 50,87	R$ 1.007,22	R$ 1.058,09	R$ 4.080,11
9	R$ 4.080,11	R$ 40,80	R$ 1.012,29	R$ 1.053,09	R$ 3.067,82
10	R$ 3.067,82	R$ 30,68	R$ 1.017,41	R$ 1.048,09	R$ 2.050,40
11	R$ 2.050,40	R$ 20,50	R$ 1.022,59	R$ 1.043,09	R$ 1.027,81
12	R$ 1.027,81	R$ 10,28	R$ 1.027,81	R$ 1.038,09	-R$ 0,00

Source: Prepared by the authors(2021)

Constructed under the same conditions as the previous tables, Table 4 shows the evolution of a R$12,000.00 debt until its settlement in the 12th month, under a monthly interest rate of 1%. As mentioned earlier, for each month n, the installments are calculated as the arithmetic mean of the installments corresponding to the SAC and Price Table systems. For example, in the first three months, we have:

- $p_1 = \frac{1120{,}00+1066{,}19}{2} = R\$1093{,}09;$
- $p_2 = \frac{1110{,}00+1066{,}19}{2} = R\$1088{,}09$ e
- $p_3 = \frac{1100{,}00+1066{,}19}{2} = R\$1083{,}09.$

Once the remaining installment values are calculated following the same procedure as in the previous examples—the other variables involved in this modality evolve similarly to those in the SAC system. In this context, we highlight the total amount paid in this transaction: R$12,787.11, obtained by summing all the installments. This amount is higher than that of the SAC (R$12,780.00) and lower than that of the Price Table (R$12,794.08).

The SACRE system is used in the settlement of certain real estate financing agreements, such as those under the Housing Finance System (SFH). Regarding its use, Santos (2015) notes as a

disadvantage the fact that the initial installments are slightly higher than those of the French system, although more affordable than those of the SAC. On the other hand, the total amounts paid under the SACRE system are lower than those of the Price Table, due to the higher amortizations that occur in the early months of the contract.

Given the amortization systems presented in this text with their specific features, applications, and inherent mathematical content—we reaffirm the focus of this article on the teaching of these systems to students in Basic Education. Accordingly, the next section presents our considerations on teaching the topic of Loans & Financing in the High School classroom, based on amortization systems.

Amortization Systems in High School Education

In the discussions presented in the previous section, we highlighted references to the formulas for simple and compound interes, $M = V \cdot (1 + n \cdot i)$ e $M = V \cdot (1 + i)^n$, as well as general formulas for the n-th term of arithmetic and geometric progression, $a_n = a_1 + r \cdot (n - 1)$ e $a_n = a_1 \cdot q^{n-1}$, and the expressions for the sum of the terms in these sequences:, $S_n = (a_1 + a_n) \cdot \frac{n}{2}$ e $S_n = a_1 \cdot \left(\frac{q^n - 1}{q - 1}\right)$. In high school textbooks, these equations appear in chapters on Financial or Commercial Mathematics and on Sequences and Series, which may be addressed in different school years depending on the author's instructional approach.

According to an analysis published by Pereira and Couto (2017) on the Financial Mathematics content found in major textbook collections intended for high school students, it is common to find discussions on simple and compound interest including comparisons between the two as well as activities involving arithmetic and geometric progressions in financial contexts. The titles analyzed form,

- Matemática: ciência e aplicações, 6th edition, by GelsonGelson Iezzi *et al*;
- Matemática: volume único, 1st edition, by Manoel Paiva;
- Matemática: ciência, linguagem e tecnologia, 1st edition, by de Jackson Ribeiro e
- Matemática: Contexto e aplicações, 6th edition, by Luiz Roberto Dante.

In some of the textbook collections mentioned above, the formulas for simple and compound

interest are associated with arithmetic and geometric progressions, respectively, or alternatively, with linear and exponential functions, in that order. However, these materials make no reference to amortization systems, whether in the chapters on Financial Mathematics or on Sequences and Series. Furthermore, few publications foster discussions on the advantages of paying in full, on nominal and effective interest rates, or on selecting the most appropriate amortization system for installment purchases. These are essential discussions within the scope of Financial Education that, in the final stage of Basic Education, should equip students to make financial decisions critically and wisely.

Conversely, our research points to the potential of working with amortization systems as a new topic in the High School classroom, under the title *Loans & Financing*. As discussed in the previous section, understanding this topic involves knowledge of "Interest" and "Progressions," which are part of the final stage of Basic Education. According to Pereira and Couto (2017), these two subjectspresent in all the textbook collections they analyzed constitute prerequisites for approaching the subject at hand.

In this context, where the topic of Amortization Systems is not commonly addressed in high school textbooks, the development of suitable approaches for this educational stage has been the focus of various studies. Among these, we highlight dissertations from the *Professional Master's Program in Mathematics in the National Network* (PROFMAT), whose repository contains 19 works related to the teaching of *Loans & Financing*. Based on the program's objective of promoting improvements in Basic Education, PROFMAT stands out as an important locus of scientific and academic production in the field of Mathematics Education.

Submitted to a systematic review process, Freitas et al. (2021) categorized the aforementioned works according to the mathematical content addressed, the teaching resources employed in the instruction of Amortization Systems, and the materials/products developed through the research. In the first category, the topics of "Interest" and "Uniform Series" are emphasized as foundational for the study of SAC, the Price Table, and other financing modalities. Regarding the resources used, table completion and graph construction are widely explored to analyze the characteristics of each system as demonstrated in the previous section of this article. For this purpose, nine authors make use of digital tools such as GeoGebra and Excel. Moreover, these same tools appear in the analyzed materials/products, ranging from the development of didactic sequences and activity sets designed for the high school classroom to the creation of financing simulators developed with the

aforementioned software.

The authors of the aforementioned dissertations address the relevance of the topic *Loans & Financing*, or *Amortization Systems*, and emphasize the need for it to be incorporated into high school classrooms. For each system, the works offer considerations regarding its use, characteristics, and the calculation of installments. However, few of these studies raise discussions on themes such as necessity and desire; planning and goals; dreams and assets. Addressing these factors constitutes an approach that is intrinsic to Financial Education.

In light of the above, we emphasize the need for instructional material that addresses the topic of *Loans & Financing*, both in the context of the discussions specific to Financial Education and in terms of mathematical content appropriate for the high school classroom. From this perspective, the following section presents the development of an educational product, the outcome of the master's research reported in this article.

Loans & Financing: The Educational Product

Presented by Freitas and Moreira (2020 and 2021a), the educational product developed during this master's research is titled *The Mathematics of Loans & Financing in High School*. It is a digital book designed to guide Basic Education teachers in presenting the two main amortization systems discussed in this article: SAC and the Price Table. The material consists of five didactic sequences to be implemented with high school students in the classroom, either in parallel with or independently from the textbook adopted by the school.

Through the proposed didactic sequences, students will be able to: establish connections between interest and progressions; identify and generalize arithmetic and geometric patterns in installment purchases; compare amortization systems by analyzing their advantages and disadvantages; calculate installment amounts whether fixed, decreasing, or prepaid; and finally, calculate the outstanding balance and total amount paid over the course of a financing agreement. To meet these objectives, the material is structured into modules, each divided into sections, as shown in Table 2.

The initial suggestion is that each module presented in Table 2 be addressed in a 100-minute session (equivalent to two class periods), preceded by an initial 50-minute session to introduce the

project. For implementation, each didactic sequence is organized into sections intended either for the teacher or for the student.

Box 2 – Chapters and Structure

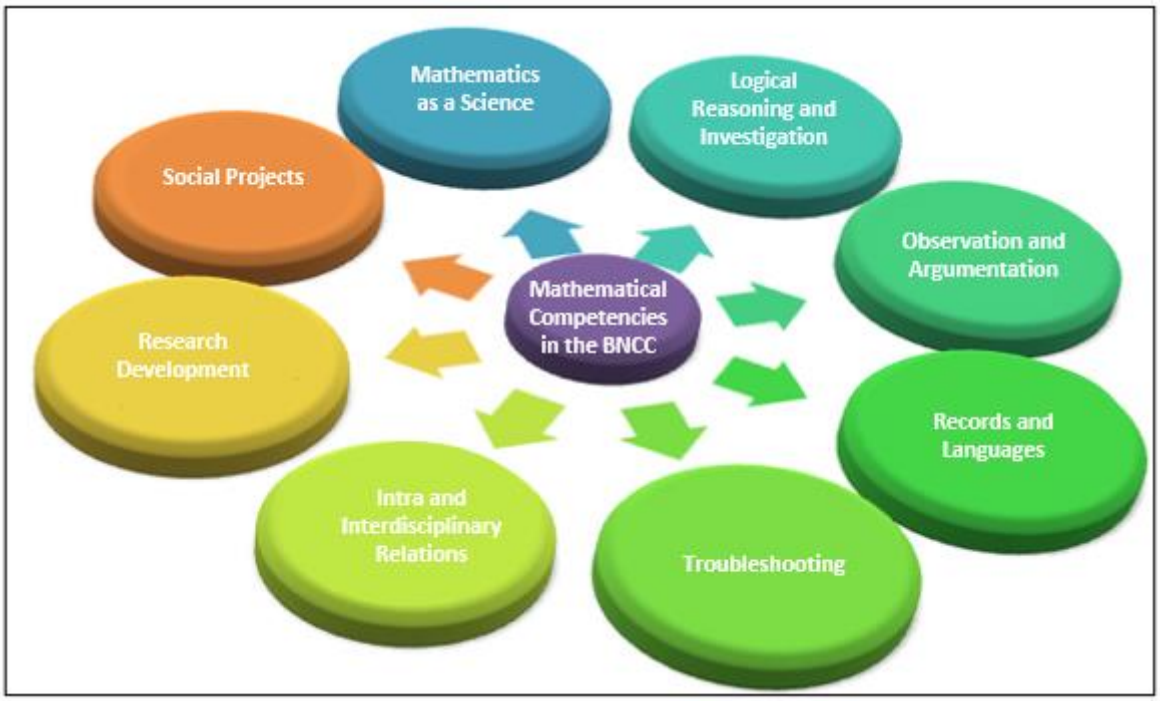
Chapter	Structure	Objective
1	Interest and Progressions	Associate the expressions of amounts in simple (or compound) interest systems with the general term formulas of arithmetic (or geometric) progressions.
2	Types of Financing	Present the concept of amortization and identify patterns through progressions and recurrences in various financing processes.
3	PRICE Table	Identify the geometric progressions formed by the updated values of overdue payments and calculate the fixed installment amount.
4	SAC System	Identify the evolution of SAC amounts as arithmetic progressions and compare debt repayment in the SAC and PRICE systems.
5	Recurrences	Explore the processes of constructing functions that verify the recurrence relations mentioned in the previous chapters.

Source: Freitas e Moreira (2020 Adaptada)

To support the teacher, the material includes guidelines for conducting each session, along with the sections titled *Research Problems*, which address the explanation of the content. In addition, we highlight the sections titled *Teacher, what if...?*, which, although not listed in Table 2, appear at the end of each didactic sequence and present possible student questions related to the topics covered. Students, in turn, are responsible for completing the introductory activities in the *Time to Explore!* section and the reinforcement questions in the *Let's Practice!* section, as well as reading the texts in *Why Didn't Anyone Tell Me This Before?*

Beyond fulfilling the previously mentioned objectives, the proposed sequences are designed to promote and reinforce specific competencies established by the BNCC for the field of Mathematics and its Technologies. According to the authors' analysis, the development and partial or full consolidation of these competencies presented in Figure 3 serve as one of the criteria for evaluating the educational product.

Figure 3 - Mathematical Competencies in the BNCC



Source: Freitas e Moreira (2021a)

In the introductory situations of the *Time to Explore!* section, students are expected to apply numerical knowledge to complete tables and, from them, identify possible patterns. To do so, the competency *Logical Reasoning and Investigation* is required, as completing the tables follows a logical sequence of operations, and pattern recognition is based on the analysis of numerical sequences. These patterns must then be described in both textual and algebraic language, calling upon the competency *Representations and Languages*.

The knowledge required to manipulate the tables, identify patterns, and record them spans the topics *Percentages & Interest*, *Sequences & Progressions*, and *Equations & Functions*. Since these topics can be approached through different thematic units (Numbers, Algebra, and Quantities/Measurements), the intradisciplinary nature of Mathematics is highlighted, reinforcing the development of the competency *Intra-/Interdisciplinary Connections*.

The algebraic language used to record the patterns identified in the tables gives rise to functions that describe the relationship between financial and temporal variables. Students are guided to test

the validity of these relationships using previously recorded values and to apply them in obtaining new values, which are subject to analysis and comparison. The act of analyzing the resulting figures and recording conclusions at the end of each situation strengthens students' *Observation and Argumentation* skills, thus evoking the corresponding competency. Furthermore, with regard to validating the derived functions, the responsibility for ensuring mathematical accuracy lies with the teacher during the explanation of the *Research Problem*. These verifications, combined with investigation and generalization, help students perceive Mathematics as a Science.

As for the development of the *Problem Solving* competency, the reinforcement questions in the *Let's Practice!* section present real-life financial contexts installment purchases, payment of installments, deadlines, returns, interest, and inflation. These topics are expanded in the texts of the *Why Didn't Anyone Tell Me This Before?* section, which relate the knowledge acquired to useful information and guidance relevant to students' Financial Education.

In this regard, it is worth noting that the texts in the aforementioned section contribute to students' introduction to the world of money and to their understanding of finance and economics. This approach aligns with the principles of School-Based Financial Education and allows us to characterize the educational product as part of a *Social Impact Project*, given the inclusive nature of the proposed topic.

Moreover, since this material is intended as a suggestion for how to address Amortization Systems in the high school classroom, it does not exhaust the topic of *Loans & Financing*. As such, various questions will arise during its implementation either from teachers encountering the subject for the first time or from students during lessons. In this sense, the questions recorded in the *Teacher, what if...?* section, along with other possible inquiries, underscore the need for further research, positioning the present work as a catalyst for the *Research Development* competency.

Having described the educational product and detailed its structure in light of the BNCC competencies specific to the area of Mathematics and its Technologies, we now turn to its implementation in two private schools in Belo Horizonte, Minas Gerais, where the master's student was employed as a teacher.

The first implementation took place in July 2020 with a 1st-year high school class (1st EM), and the second in November of the same year with a 3rd-year class (3rd EM) at a different school.

Due to the suspension of in-person classes from March until the end of the academic year caused by the spread of the coronavirus—both implementations were conducted in virtual learning environments, where participants interacted via videoconferencing platforms: Google Meet and Zoom, respectively.

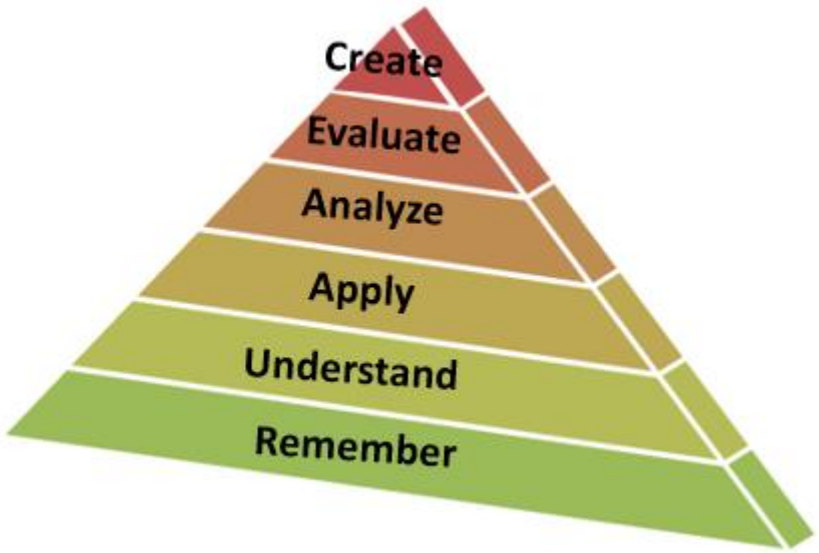
Right from the first session, which introduced the project, participants were struck by the comparison between the cash price of a car and the total amount to be paid for the same vehicle through financing a situation similar to the story presented in the introduction of the digital book. On that occasion, some students remarked that it would be better to save and pay in full, while others suggested researching better financing options. These suggestions served as references for the following classes and were incorporated into the explanations in the *Research Problem* sections. Another point that surprised students in both groups was the absence of topics related to this subject in the textbooks adopted by their schools.

As for the activities proposed in the *Time to Explore!* and *Let's Practice!* sections, the students' questions were similar to those expected in face-to-face classes. The topics addressed in these activities, as well as in the texts from *Why Didn't Anyone Tell Me This Before?*, sparked valuable discussions that significantly contributed to the Financial Education of those involved.

Students concluded their participation in the project by offering critiques and suggestions for improving the educational product. Among the suggestions, we highlight the need for increased class time per module, particularly when the material is implemented in remote learning environments. These classroom applications also proved useful for refining the material—such as adding new teacher instructions, additional questions to the *Teacher, what if...?* section, and revising an item in the *Let's Practice!* section.

Following implementation, the educational product was submitted for analysis and validation, using as evaluation criteria the aforementioned BNCC competencies for the area of Mathematics and its Technologies, as well as the learning objectives of Bloom's Taxonomy. Developed by a research group led by Benjamin Bloom in 1956, this taxonomy is a classification system for educational goals. In it, the progression of knowledge is described in stages, as illustrated in Bloom's Pyramid, shown in Figure 4.

Figure 4 – Bloom's Taxonomy Pyramid



Source: Freitas e Moreira (2021a)

In order to indicate the level of difficulty associated with the content being assessed, a set of cognitive verbs is associated with each tier of the pyramid shown in Figure 4. According to descriptions by Mello et al. (2019), the first two levels, at the base of the pyramid, constitute the knowledge acquisition phase, referring to the ability to memorize and understand what is being taught. The subsequent stages, *Apply* and *Analyze*, involve the development of critical thinking, while the final two levels reflect the progression from critical thinking to the ability to create new knowledge.

In this regard, Bloom's proposed classification serves as an effective tool to support the organization and analysis of educational objectives within didactic sequences, assessments, and curricula. Therefore, the aforementioned stages, along with the set of actions associated with each, will guide the analysis of the didactic sequences proposed in each module.

Stage E1 – *Remember* requires the student to accurately recall previously covered information and content, while stage E2 – *Understand* assesses whether the learner is able to modify original information, expand (or reduce) it, and represent it in different formats. In this context, these stages serve as the starting point for Module M1 – *Interest and Progressions* in our material, as it recalls students' prior knowledge of arithmetic and geometric progressions, contextualizes them in financial situations, and links them to interest rate formulas.

Stage E3 – *Apply* requires the student to use previously learned information in new situations, selecting, transferring, and applying data and principles to solve a problem or complete a task with minimal guidance. This is precisely what occurs in Module M2 – *Types of Financing*, as the newly acquired concepts of amortization allow students to recognize patterns across different processes and characterize the financing models presented.

The specific features of amortization systems are addressed in Modules M3 – *Price Table* and M4 – *SAC System*. This exploration involves distinguishing, classifying, and relating the structural elements of each system, introduced at the beginning of the didactic sequences in the *Time to Explore!* sections. These actions correspond to Stage E4 – *Analyze* in Bloom's Taxonomy. In these modules, students are tasked with evaluating, comparing, discussing, and explaining the advantages and disadvantages of the systems studied. Such tasks are aligned with the cognitive verbs associated with Stage E5 – *Evaluate*.

As for the highest level, E6 – *Create*, students are expected to combine elements of knowledge to form a coherent Whole one that was not previously evident. This occurs in Module M5 – *Recurrences*, whose goal is to explore the construction processes of functions that define the recurrence relations introduced in previous modules. It is worth noting that a recurrence describes the behavior of a numerical sequence by relating consecutive terms in that sequence. Although many such relations are not classified as arithmetic (or geometric) progressions, concepts linked to the sum of terms in these sequences are useful for deriving the functions that solve them. Thus, in M5, we witness the creation of new knowledge (*Recurrences*) from prior knowledge (*AP/GP*).

Having described the progression of knowledge on the topic of *Loans & Financing* as developed in the didactic sequences of our educational product, the analyses presented here support

the validation of this material in light of the educational objectives defined by Bloom's Taxonomy.

Final considerations

Having presented our reflections on the importance of Financial Education for society as a whole and the need for its implementation in schools, we recognize the topic *Loans & Financing* as a matter of great relevance, as the knowledge it encompasses directly influences decision-making in installment purchases. These are forms of knowledge that support budget management and future planning, while also helping to maintain a healthy economy for the individual and their family.

Consumers' lack of awareness regarding the advantages and disadvantages of the various credit options available on the market can lead to consequences that undermine the economic capacity of society at large. In this sense, discussions on the topic of *Loans & Financing* should not be limited to finance-related courses at the technical, undergraduate, or graduate level, but should be accessible to all citizens who, when unable to pay in full, may opt consciously and responsibly for installment strategies.

With this in mind, and aiming to form a generation of consumers who know how to use credit lines responsibly and conscientiously, we advocate for the inclusion of the topic *Loans & Financing* as part of the high school curriculum, within the themes outlined for Financial Education. To this end, the educational product *The Mathematics of Loans & Financing in High School*, presented in this article, constitutes an effective tool, as demonstrated by the analyses provided.

Based on mathematical content specific to high school, Freitas and Moreira (2021b) offer reflections on the main amortization systems, highlighting their features and applications. With an approach that emphasizes student protagonism and the mediating role of the mathematics teacher, the material fosters discussions that are highly relevant to participants' Financial Education, while also developing the competencies set forth in the BNCC and deepening students' mathematical understanding in accordance with the progression outlined in Bloom's Taxonomy.

Thus, we hope that the knowledge and skills developed through this educational product will contribute to shaping a new generation of consumers who, equipped with an understanding of the main amortization systems, will be able to argue with awareness, negotiate critically, and make responsible decisions when facing the need for installment purchases.

Finally, we conclude this article with the hope of presenting mathematics teachers and researchers—along with professionals in related Fields with an educational product of significant value for the development of Financial Education among both educators and students. We therefore invite readers to download the digital book free of charge at https://sbm.org.br/wp-content/uploads/2021/11/A_Matematica_dos_Emprestimos_e_Financiamentos.pdf and, through it, deepen their understanding of the topic *Loans & Financing*.

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